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TURBULENT TRANSFER MECHANISM AND SUSPENDED SEDIMENT IN CLOSED CHANNELS

By Hassan M. Ismail, Assoc. M. ASCE

HYDRAULICS DIVISION

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PAPERS

TURBULENT TRANSFER MECHANISM AND SUSPENDED SEDIMENT IN CLOSED CHANNELS

By Hassan M. Ismail,[1] Assoc. M. ASCE

Synopsis

Tests on a rectangular channel, 10.5 in. by 3 in. in cross section, transporting water mixed with different proportions of sand, are described in this paper. In terms of mean sedimentation diameters, two sizes of sand were used—0.10 mm and 0.16 mm. The experimental data were used to study (*a*) the effects of the presence of sand in suspension on the characteristics of the flow, and (*b*) the momentum transfer coefficient and the concentration transfer coefficient at the profile through the center line of the channel, assuming a two-dimensional flow.

The present experiments show that the universal constant of turbulent exchange, k, decreases with the increase of the suspended material. The value of k decreases to 0.20 when the average concentration is 4.3% by weight. No limit was approached for the decrease of k. The friction coefficient λ was hardly affected by the presence of the sand when the velocity of flow was higher than a certain rate. This velocity is related to the bed load; it is the velocity at which all the dunes on the bed are carried in suspension; and it increases with the total load in the flume and the size of the sand used. Below that velocity, λ exceeds that of clear water.

The sediment transfer coefficient ϵ_s was found to be equal to 1.5 times the momentum transfer coefficient, ϵ_m, for the 0.10-mm sand and to 1.3 ϵ_m for the 0.16-mm sand. Both ϵ_s and ϵ_m follow the normal parabolic form at the outer two thirds of the channel and have a constant value at the middle third.

Introduction

Notation.—The letter symbols introduced in this paper are defined where they first appear, in the text or by illustration, and are assembled alphabetically in the Appendix for convenience of reference.

The modern developments in the turbulent flow theory have created interest in the sediment suspension and the heat flow problems. It was Morrough P.

Note.—Written comments are invited for publication; the last discussion should be submitted by August 1, 1951.

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O'Brien,[2] M. ASCE, who first made use of the Taylor-Schmidt findings in deriving the equation for the distribution of suspended sand in a turbulent flow. By equating the rate of upward transfer, $-\epsilon_s \frac{dC}{dy}$, of suspended sediment resulting from turbulent exchange and the rate of settling, $> w\,C$, under gravitational force, Mr. O'Brien obtained the equation:

$$\epsilon_s \frac{dC}{dy} + w\,C = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which y is measured vertically upward; C is the concentration of suspended material; w is the settling velocity of the sediment in the still fluid; and ϵ_s is the transfer coefficient for the sediment.

Mr. von Kármán,[3] Hon. M. ASCE, gave the analogy between the transfer of mass or of heat and the transfer of momentum. He showed that the coefficient of all these kinds of transfer should have the same form, but he stated that it was subject to further discussion as to whether those coefficients have the same values.

Another step was taken by T. K. Sherwood and B. B. Woertz[4] who found that the transfer coefficient for water vapor in a turbulent gas stream was not equal to the momentum transfer coefficient ϵ_m, but that they seemed to bear a constant relation to each other. Other investigators disclosed that this approximation can be assumed also in case of heat transfer. Therefore, it may be assumed that ϵ_s can be related linearly to ϵ_m:

$$\epsilon_s = \beta\,\epsilon_m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which β is the coefficient of proportionality. It is known that, for turbulent flow, $\tau = \rho\,\epsilon_m\,du/dy$, in which τ is the shearing stress at any point; u is the velocity; and ρ is the density of the fluid. For two-dimensional flow, $\tau = \tau_o\,(1 - y/y_m)$; in which τ_o is the shearing stress at the boundary; and y_m is the vertical distance to the maximum velocity—that is the plane of zero shear. Thus, $\epsilon_m = \frac{\tau_o\,(1 - y/y_m)}{\rho\,du/dy}$. The von Kármán universal velocity defect law[5] is

$$\frac{u - u_{\max}}{U_f} = \frac{1}{k} \log_e \frac{y}{y_m} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which $u_{\max}$ is the maximum velocity and $U_f = \sqrt{\tau_o/\rho}$. Therefore, when du/dy is evaluated from Eq. 3, the result is

$$\epsilon_m = k\,U_f\,y\left(1 - \frac{y}{y_m}\right) \ldots\ldots\ldots\ldots\ldots\ldots (4a)$$

---

[2] "Review of the Theory of Turbulent Flow and Its Relation to Sediment-Transportation," by Morrough P. O'Brien, *Transactions*, Am. Geophysical Union, Vol. 14, 1933, pp. 487–491.

[3] "Some Aspects of the Turbulence Problem," by Theodor von Kármán, *Mechanical Engineering*, Vol. 57, 1935, pp. 407–412.

[4] "Mass Transfer Between Phases," by T. K. Sherwood and B. B. Woertz, *Industrial & Engineering Chemistry*, Vol. 31, 1939, p. 1034.

[5] "The Mechanics of Turbulent Flow," by Boris Bakhmeteff, Princeton Univ. Press, Princeton, N. J., 1936.

or

$$\epsilon_s = \beta\, k\, U_f\, y \left(1 - \frac{y}{y_m}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4b)$$

and, in its dimensionless form,

$$\frac{\epsilon_s}{\beta\, k\, U_f\, y_m} = \frac{y}{y_m}\left(1 - \frac{y}{y_m}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The universal constant k is used on the left-hand side so that the right-hand side will be a function of only the position.

Eqs. 4 give zero values for both ϵ_m and ϵ_s at the center of the channel. Many investigators maintain that the transfer coefficients are not zero at the center and, therefore, that the equations are not valid at this point. Mr. von Kármán[6] explained how these relations were derived from the assumption of similar flow pattern; then he stated that in the center part of a pipe the similarity assumption cannot be correct. F. A. Brooks and W. Berggren[7] showed how L. Prandtl tried to correct that region and how the experiments of J. Nikuradse gave definite values of ϵ_m at the center of the pipes in his experiments. They showed that the two reasonable assumptions which can be made at the center are either to take ϵ_m as a constant according to the results of Messrs. Sherwood and Woertz[4] or to assume an error curve for ϵ_m.

If ϵ_s is constant over a certain region, Eq. 1 can be integrated for that region to give

$$\log_e \frac{C}{C_a} = \frac{w}{\epsilon_s}(y - a) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

in which C_a is the concentration at any arbitrary reference level $y = a$.

If ϵ_s follows Eq. 4*b* in another region, Eq. 1 will be integrated to give

$$\log_e \frac{C}{C_a} = \frac{w}{\beta\, k\, U_f} \log_e \left(\frac{y_m - y}{y} \times \frac{a}{y_m - a}\right) \ldots\ldots\ldots\ldots (7a)$$

or

$$\frac{C}{C_a} = \left(\frac{h}{h_a}\right)^z \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7b)$$

in which $z = \dfrac{w}{\beta\, k\, U_f}$ and $h = \dfrac{y_m}{y} - 1$.

Vito A. Vanoni,[8] M. ASCE, verified Eq. 7*a* and showed how well it fitted the experimental points in an open channel. He also showed that the existence of a suspended load tends to suppress or damp out the turbulence, causing a decrease in the value of k, the von Kármán universal constant of turbulent exchange (Eq. 3). The main lines of Mr. Vanoni's work were extended in the present paper to cover the study of a closed channel.

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[6] "Turbulence and Skin Friction," by Theodor von Kármán, *Journal of the Aeronautical Sciences*, Vol. 1–2, 1934, pp. 1–20.

[7] "Remarks on Turbulent Transfer Across Planes of Zero Momentum-Exchange," by F. A. Brooks and W. Berggren, *Transactions*, Am. Geophysical Union, Pt. VI, 1944, pp. 889–896.

[8] "Transportation of Suspended Sediment by Water," by Vito A. Vanoni, *Transactions*, ASCE, Vol. 111, 1946, pp. 67–133.

Objective of Experiments

The two main objectives of the present experiments are as follows:

1. The first objective was to continue the study of the effects of the suspended sediment on the characteristics of the flow. A closed channel was selected for these studies instead of an open channel because of the relative ease with which steady uniform conditions can be established and accurate measurements made. An attempt was made to solve the partial differential equation of the second degree that represents the sediment distribution in the case of a circular pipe, but the equation was too complicated to be solved in reasonable time. As a result the flow between two wide parallel plates was used. In order to study the effect of the presence of sand on the wall roughness all the walls of the flume were painted to make them as smooth as possible.

2. The second objective was to study both the momentum transfer coefficient ϵ_m and the sediment transfer coefficient ϵ_s, their distribution over the cross section, especially near the center line of the channel, and their interrelation.

Apparatus and Procedure

Main Flume.—The experiments were conducted in a closed-circuit flume (Fig. 1) about 10.5 in. wide, 3 in. deep, and 40 ft long. Precise measurements show that the section is 2.986 in. ($\pm$ 0.5%) by 10.507 in. ($\pm$ 0.4%). For the present experiments the flume was kept horizontal.

The top cover of the flume was made of $\frac{5}{16}$-in. steel plate, supported from above, as shown in Fig. 1(*c*). The cover was sealed at the sides by a solid rubber rod ($\frac{3}{16}$ in. in diameter). All the surfaces were covered carefully with two coats of bitumastic paint which gave a reasonably smooth surface.

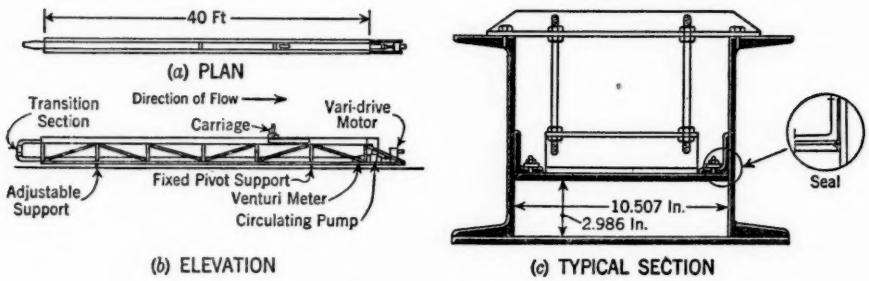


Fig. 1.—Layout and Dimensions, 10-In. Closed Circuit Flume

The pitot tube and the samplers can traverse the cross section through an opening in the flume cover (Fig. 2). This working section is 28.5 ft from the entrance to the channel. On the downstream side of the working section, there were glass windows 5 ft long on each side wall and a plastic window in the cover 10.5 in. long and 6 in. wide. The water or the mixture of sand and water was circulated in the flume by a propeller pump driven by a variable speed motor.

Measuring Equipment.—A standard Prandtl pitot-static tube $\frac{1}{4}$ in. in diameter was used to measure the velocity. The differential pressure on the tube was read to 0.001 ft on a water-air manometer and converted to velocity by

assuming a coefficient of unity for the tube. Two samplers were used to determine the suspended sand concentration at different points. One of them reached the upper surface and the other reached the bed (see Fig. 3). Both were made of brass tubing with an external diameter of $\frac{3}{16}$ in. Their tips were flattened so that the inside dimensions were 0.22 in. by 0.05 in. Each sample was siphoned from the flow and collected in a bottle of 1-liter volume.

The elevation of the bottle and, hence, the head on the siphon could be adjusted to give the same velocity at the inlet to the sampler as the local stream velocity at the sampling point. The effect of the rate of sampling on the proportion of sediment removed with the sample, as well as the sampling technique, has been clearly explained elsewhere.[8,9] The sediment was removed from the sample by filtering and was dried and weighed to the nearest 0.005 g.

A general sample was collected by a special sampler at the end of the flume. Three liters were collected for each run and they were used to find an average value of concentration of suspended sand over the section. This average concentration was called $\bar{c}$.

The pitot tube, or one of the two samplers, could be fixed to a carriage contrived so that it could move across and along the stream and in the vertical direction. The three coordinates of the instrument on the carriage could be read from scales that enabled the operator either to locate the position of the instrument or to set it at a predetermined position.

The flow rate and the average velocity, U, in the channel were determined from velocity measurements with the pitot tube. From complete velocity traverses in runs 1, 2, 5, 6, 7, and 71, the relation—

$$U = 0.95\, u'' \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

—was found, in which u'' is the average velocity along the center-line profile. In all runs, except the aforementioned, the average velocity was determined from this empirical relation. This method was adopted in preference to that of using the venturi meter. The meter behaved erratically, probably because of the presence of sediment in the flow and because of the disturbances from the pump, which is close to the meter.

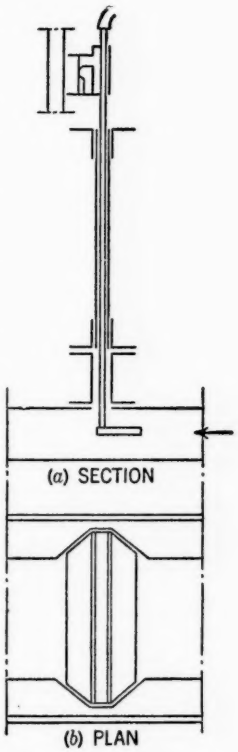


FIG. 2.—THE WORKING SECTION

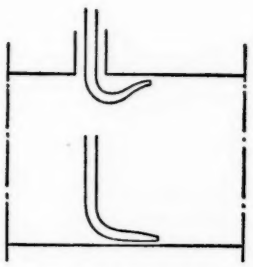


FIG. 3.—SAMPLERS

The head lost in the flume was measured by four pressure taps connected at 9.8 ft, 18.8 ft, 28.25 ft, and 33.25 ft from the inlet. Settling pots were included

[9] "Laboratory Investigations of Suspended Sediment Samplers," *Report No. 5*, U. S. Engineer District Suboffice, St. Paul, Minn., December, 1941.

in all pressure lines to trap sediment and prevent it from lodging in the lines. The static head could be read to the nearest 0.05 in. The temperature of the fluid was measured by a thermometer at the end of the flume.

It was very helpful that the same sand which was used by Mr. Vanoni[8] was still available. The sands with sedimentation diameters of 0.10 mm and 0.16 mm were used in this case. Their size distributions were checked by sieve analyses and were found to be still the same. Therefore, his measured values of the settling velocity w were used in the present work. The size distribution is given in Fig. 4.

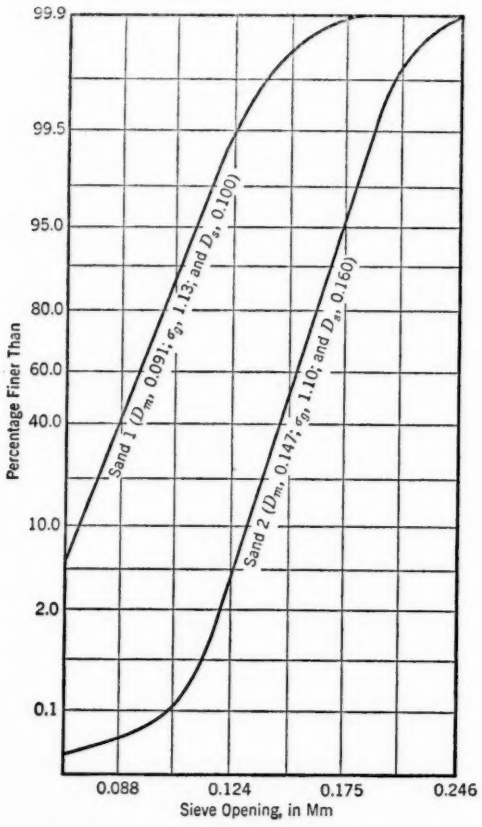


FIG. 4.—SIZE DISTRIBUTION OF SAND

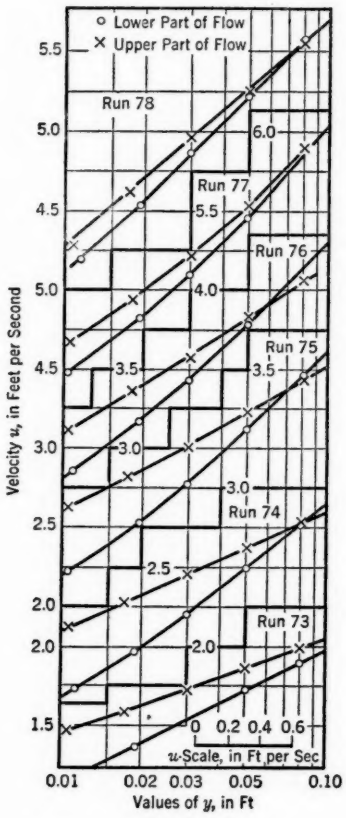


FIG. 5.—VELOCITY PROFILES

Procedure.—A complete series of experiments was made for each of the two sand sizes. Each series had two variables, the total weight of sand that existed in the flume and the velocity of the flow. Beginning with clear water, 50 g of sand were added; the weight added each time was about the same as that already in the system, following more or less the terms of a geometric series. In general, for each sand content, experiments were made with four flow velocities. They were about 1.7 ft per sec, 2.6 ft per sec, 3.5 ft per sec, and 4.6 ft per sec.

Results

A sample of the basic experimental data is given in Figs. 5, 6, and 7. Fig. 5 gives the semilogarithmic plot of the velocity profiles along the center line of the channel; Fig. 6 gives log C versus log h; and Fig. 7 gives log C versus y. The results obtained from all the experiments are collected and summarized in Table 1.

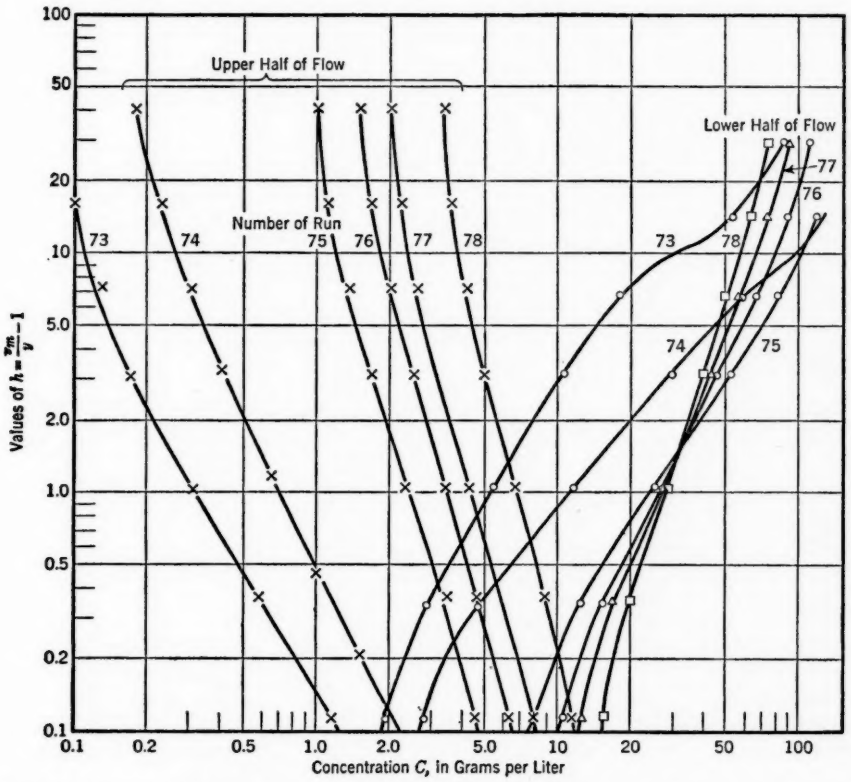


Fig. 6.—Measured Sediment Distribution at Center of Channel Plotted Against $h = y_m/y - 1$

It is to be noted that, except for the study of the central part of the channel, the flow was divided into two parts, one part covered nearly the upper half and the other part covered the lower half. Each part was treated alone as if it were a completely separate flow. This was studied and proved to be true by H. Schlichting.[10] H. A. Einstein,[11] Assoc. M. ASCE, divided the section of open trapezoidal channels into three parts, each corresponding to one of the sides, and he assumed that the roughness of each wall would control the flow in the corresponding part.

[10] "Experimental Investigation of the Roughness Problem," by H. Schlichting, *Proceedings*, ASCE, November, 1937, p. 16 following p. 1834.

[11] "Formulas for the Transportation of Bed Load," by H. A. Einstein, *Transactions*, ASCE, Vol. 107, 1942, p. 575.

TABLE I.—SUMMARY

Run	Mean velocity, U	Hydraulic gradient, i	Total sand load, W	Concentration, $\bar{c}$	Temperature, t	Viscosity, ν	Settling velocity, w	Friction velocity, U_f	Depth, y_m	Friction Velocity	
										Top, U_{tf}	Bottom U_{bf}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	1.78	0.00244	0	....	15.9	1.26	....	0.0883	....	....	....
2	2.45	0.00452	0	....	18.0	1.21	....	0.12	....	....	....
3	3.00	0.00587	0	....	19.3	1.17	....	0.1367	....	....	....
4	3.00	0.00687	0	....	17.9	1.30	....	0.148	....	....	....
5	3.66	0.01026	0	....	18.0	1.28	....	0.181	....	....	....
6	4.38	0.013	0	....	18.0	1.20	....	0.204	....	....	....
7	5.08	0.0172	0	....	19.5	1.16	....	0.234	....	....	....
8	1.78	0.00257	50	....	21.4	1.15	....	0.0906	....	....	....
9	2.41	0.0045	50	....	15.6	1.26	....	0.12	....	....	....
10	3.06	0.0067	50	....	21.7	1.10	....	0.146	....	....	....
11	3.70	0.0093	50	....	22.0	1.10	....	0.172	....	....	....
12	4.40	0.0128	50	....	20.7	1.11	....	0.202	....	....	....
13	5.12	0.0167	50	....	23.9	1.05	....	0.231	....	....	....
14	1.17	0.0011	100	....	21.3	1.10	....	0.0593	....	....	....
15	2.01	0.00312	100	....	25.2	1.05	....	0.0998	....	....	....
16	2.37	0.00404	100	0.015	21.0	1.11	....	0.1135	....	....	....
17	2.39	0.00413	100	....	22.8	1.08	....	0.115	....	....	....
18	3.53	0.00863	100	0.025	17.0	1.22	....	0.116	....	....	....
19	1.74	0.00248	150	....	16.0	1.26	....	0.089	....	....	....
20	2.56	0.00477	150	0.015	19.0	1.19	....	0.1233	....	....	....
21	2.60	0.00486	150	....	20.0	1.15	....	0.1246	....	....	....
22	3.58	0.00881	150	0.033	20.7	1.14	....	0.168	....	....	....
23	4.93	0.0155	150	0.057	20.8	1.14	....	0.222	....	....	....
24	1.12	0.0011	200	....	19.0	1.16	....	0.0593	....	....	....
25	1.74	0.00239	200	0.01	19.0	1.16	....	0.0873	....	....	....
26	2.40	0.0045	200	0.045	20.9	1.12	....	0.120	....	....	....
27	3.29	0.00744	200	0.07	19.0	1.16	....	0.154	....	....	....
28	4.23	0.0115	200	0.05	21.8	1.10	....	0.1915	....	....	....
29	5.28	0.0179	200	0.057	20.5	1.13	....	0.239	....	....	....
30	1.76	0.00239	290	0.02	22.5	1.07	....	0.0873	....	....	....
31	2.39	0.00422	290	0.035	18.5	1.18	....	0.116	....	....	....
32	3.26	0.0078	290	0.06	18.7	1.18	....	0.158	....	....	....
33	4.25	0.01191	290	0.065	19.3	1.17	....	0.195	....	....	....
34	1.73	0.00238	390	0.02	20.5	1.13	....	0.0872	....	....	....
35	2.65	0.00504	390	0.06	17.6	1.21	....	0.127	....	....	....
36	3.58	0.00835	390	0.08	21.6	1.10	....	0.163	....	....	....
37	4.97	0.0151	390	0.11	20.0	1.15	....	0.2196	....	....	....
38	1.73	0.00229	490	0.035	21.0	1.12	0.0267	0.0855	....	....	....
39	2.60	0.00449	490	0.08	17.5	1.22	0.025	0.12	....	....	....
40	3.60	0.0085	490	0.115	18.7	1.19	0.0255	0.165	....	....	....
41	4.89	0.0147	490	0.16	20.0	1.15	0.0262	0.216	....	....	....
42	1.08	0.00101	600	....	18.0	1.2	....	0.0567	0.13	0.0554	0.057
43	1.56	0.00193	600	0.02	18.2	1.2	0.0252	0.0785	0.124	....	....
44	1.72	0.00229	600	0.015	20.6	1.13	0.0265	0.0855	0.124	....	....
45	2.37	0.00413	600	0.06	21	1.12	0.0267	0.1148	0.124	....	....
46	2.96	0.00582	600	0.12	23.4	1.05	0.028	0.1363	0.124	....	....
47	3.59	0.0084	600	0.15	24.4	1.02	0.0285	0.163	0.124	....	....
48	4.69	0.0142	600	0.17	23.7	1.05	0.0281	0.212	0.124	....	....
49	1.18	0.00116	800	....	19.2	1.16	....	0.0608	0.122	0.0612	0.0604
50	1.83	0.0025	800	....	19.2	1.16	....	0.0893	0.124	....	....
51	1.75	0.00227	800	0.055	24.8	1.02	0.0287	0.0851	0.124	....	....
52	2.65	0.00499	800	0.165	21.2	1.14	0.0268	0.126	0.124	....	....
53	3.58	0.0086	800	0.26	21.0	1.12	0.0267	0.165	0.124	....	....
54	4.91	0.0148	800	0.34	25.5	1.01	0.0291	0.217	0.124	....	....
55	1.71	0.00224	1,200	0.175	25.8	1.01	0.0292	0.0845	0.124	....	....
56	2.58	0.00457	1,200	0.47	20.5	1.14	0.0264	0.121	0.122	0.122	0.120
57	3.59	0.00832	1,200	0.60	23.8	1.04	0.0282	0.163	0.121	0.164	0.162
58	4.71	0.0136	1,200	0.60	25.5	1.01	0.0291	0.208	0.124	....	....
59	1.18	0.0013	2,000	....	20.5	1.14	....	0.0644	0.124	....	....
60	1.42	0.00166	2,000	....	21.0	1.13	....	0.0728	0.124	....	....
61	1.71	0.00226	2,000	0.53	21.8	1.11	0.0271	0.085	0.13	0.083	0.0865
62	2.61	0.00467	2,000	1.23	21.5	1.11	0.027	0.122	0.124	....	....
63	3.55	0.0083	2,000	1.46	23.8	1.04	0.0282	0.163	0.124	....	....
64	4.67	0.0138	2,000	1.54	21.6	1.11	0.027	0.210	0.124	....	....

[a] Dunes and streaks. [b] Not able

OF EXPERIMENTS

Universal Constant			Reynolds number, $R \times 10^{-5}$	Friction coefficient, $\lambda \times 10^2$	Ratio, $w/(kU_f)$		$z = w/(\beta k U_f)$		$\frac{\epsilon_{ss}}{\beta k y_m U_f}$	Bed condition[a]	Run
Top, k_t	Bottom, k_b	Average, k_{avg}			Top	Bottom	Top	Bottom			
(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(1)
....	....	0.370	4.35	1.92	....	....	....	....	....	....	1
....	....	0.370	6.50	1.88	....	....	....	....	....	....	2
....	....	0.374	8.51	1.64	....	....	....	....	....	....	3
....	....	0.378	6.91	1.91	....	....	....	....	....	....	4
....	....	0.375	8.70	1.92	....	....	....	....	....	....	5
....	....	0.372	11.80	1.70	....	....	....	....	....	....	6
....	....	0.373	14.65	1.67	....	....	....	....	....	....	7
....	....	0.373	5.22	2.03	....	....	....	....	....	Smooth	8
0.392	0.368	0.380	5.89	1.94	....	....	....	....	....	Smooth	9
0.369	0.365	0.367	9.80	1.79	....	....	....	....	....	Smooth	10
....	....	0.360	11.86	1.70	....	....	....	....	....	Smooth	11
....	....	0.375	13.9	1.65	....	....	....	....	....	Smooth	12
0.391	0.369	0.380	18.0	1.60	....	....	....	....	....	Smooth	13
....	....	0.388	3.76	2.00	....	....	....	....	....	Streaks	14
....	....	0.362	7.16	1.94	....	....	....	....	....	Streaks	15
....	....	0.363	7.46	1.80	....	....	....	....	....	Streaks	16
....	....	0.368	7.97	1.81	....	....	....	....	....	Streaks	17
....	....	0.358	9.21	1.73	....	....	....	....	....	Smooth	18
....	....	0.376	4.25	2.04	....	....	....	....	....	Streaks	19
0.387	0.329	0.358	6.47	1.82	....	....	....	....	....	Streaks	20
0.388	0.368	0.378	7.64	1.80	....	....	....	....	....	Streaks	21
0.368	0.346	0.357	10.70	1.72	....	....	....	....	....	Smooth	22
0.37	0.358	0.364	14.75	1.60	....	....	....	....	....	Smooth	23
....	....	0.402	3.24	2.19	....	....	....	....	....	....[a]	24
....	....	0.372	5.02	1.97	....	....	....	....	....	Streaks	25
....	....	0.370	7.42	1.95	....	....	....	....	....	Streaks	26
....	....	0.353	9.50	1.72	....	....	....	....	....	Smooth	27
0.367	0.345	0.356	13.60	1.61	....	....	....	....	....	Smooth	28
....	....	0.355	16.0	1.60	....	....	....	....	....	Smooth	29
....	....	0.365	5.90	1.93	....	....	....	....	....	Streaks	30
....	....	0.357	6.68	1.85	....	....	....	....	....	Streaks	31
....	....	0.362	9.10	1.83	....	....	....	....	....	Smooth	32
0.374	0.35	0.362	12.04	1.65	....	....	....	....	....	Smooth	33
0.378	0.365	0.371	5.30	1.99	....	....	....	....	....	Streaks	34
0.382	0.350	0.366	7.02	1.80	....	....	....	....	....	Streaks	35
0.368	0.354	0.361	11.50	1.63	....	....	....	....	....	Smooth	36
0.369	0.344	0.357	14.6	1.53	....	....	....	....	....	Smooth	37
0.371	0.334	0.353	6.00	1.92	....	....	....	....	....	Streaks	38
0.355	0.330	0.343	8.30	1.66	....	....	....	....	....	Streaks	39
0.372	0.348	0.36	11.75	1.65	....	....	....	....	....	Smooth	40
....	....	0.355	16.5	1.54	....	....	....	....	....	Smooth	41
0.368	0.366	0.367	3.5	2.17	....	....	....	....	....	Dunes	42
....	....	0.360	5.05	1.98	....	....	....	....	....	....[a]	43
0.352	0.334	0.343	5.9	1.94	....	....	....	....	....	....[a]	44
0.357	0.348	0.353	8.3	1.84	0.65	0.67	....	0.467	....	Streaks	45
0.357	0.346	0.352	10.94	1.66	0.575	0.593	0.40	0.40	....	Streaks	46
0.362	0.342	0.351	13.8	1.63	0.486	0.514	0.355	0.356	....	Smooth	47
0.366	0.346	0.356	17.38	1.62	0.366	0.388	0.198	0.262	....	Smooth	48
0.375	0.348	0.362	3.95	2.08	....	....	....	....	....	Dunes	49
0.387	0.332	0.360	6.12	1.86	....	....	....	....	....	Streaks	50
0.363	0.338	0.351	6.67	1.85	0.93	0.996	....	0.75	....	Streaks	51
....	....	0.354	9.03	1.78	0.60	0.60	....	0.46	....	Streaks	52
....	....	0.365	13.6	1.68	0.444	0.444	0.325	0.352	....	Smooth	53
0.378	0.367	0.372	18.9	1.54	0.355	0.366	0.227	0.269	....	....[b]	54
0.360	0.309	0.335	6.58	1.91	0.96	1.12	0.58	0.58	....	Streaks	55
0.375	0.320	0.348	8.8	1.72	0.577	0.69	0.438	0.438	....	....	56
0.376	0.362	0.369	13.4	1.62	0.458	0.483	0.355	0.38	....	....[b]	57
0.357	0.355	0.356	18.1	1.53	0.392	0.394	0.30	0.30	....	....[b]	58
0.41	0.37	0.39	4.01	2.33	....	....	....	....	....	....[a]	59
0.356	0.356	0.356	4.88	2.06	....	....	....	....	....	....[a]	60
0.335	0.312	0.324	6.0	1.93	....	1.003	....	0.65	0.123	Streaks	61
0.347	0.323	0.335	6.66	1.71	0.638	0.685	0.437	0.51	0.195	....[b]	62
0.354	0.338	0.346	13.2	1.64	0.489	0.512	0.37	0.385	0.197	....[b]	63
0.348	0.348	0.348	16.35	1.58	0.37	0.37	0.262	0.293	0.204	....[b]	64

to see the condition of the bed.

TABLE I.—

Run	Mean velocity, U	Hydraulic gradient, i	Total sand load, W	Concentration, $\bar{c}$	Temperature, t	Viscosity, ν	Settling velocity, w	Friction velocity, U_f	Depth, y_m	Friction Velocity	
										Top, U_{tf}	Bottom, U_{bf}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
65	1.74	0.0024	3,600	0.92	24.5	1.03	0.0285	0.0875	0.128	0.086	0.0883
66	2.61	0.00476	3,600	2.90	19.0	1.18	0.0257	0.123	0.113	0.128	0.1167
67	3.48	0.00818	3,600	3.08	21.6	1.11	0.0271	0.1617	0.12	0.164	0.158
68	4.89	0.0153	3,600	3.10	20.7	1.14	0.0265	0.221	0.124	....	....
69	1.66	0.0025	6,800	1.49	20.5	1.14	0.0265	0.0893	0.13	0.0873	0.095
70	2.61	0.00471	6,800	5.40	24.0	1.04	0.0283	0.1225	0.119	0.1248	0.117
71	3.56	0.00827	6,800	5.77	23.0	1.09	0.0278	0.162	0.118	0.166	0.155
72	4.74	0.0141	6,800	6.68	25.0	1.02	0.0288	0.212	0.130	0.207	0.217
73	1.69	0.00277	13,200	2.39	21.0	1.11	0.0267	0.094	0.136	0.0895	0.096
74	2.39	0.00415	13,200	7.23	25.0	1.02	0.0288	0.115	0.115	0.119	0.111
75	2.96	0.00591	13,200	12.57	21.6	1.11	0.027	0.1373	0.112	0.144	0.126
76	3.55	0.00827	13,200	12.65	24.5	1.025	0.0285	0.1622	0.105	0.174	0.145
77	4.18	0.01115	13,200	12.28	26.2	1.00	0.0294	0.188	0.114	0.195	0.176
78	4.91	0.01487	13,200	12.38	25.4	1.01	0.029	0.218	0.12	0.221	0.211
79	1.61	0.0029	23,200	2.92	24.1	1.03	0.0284	0.0963	0.152	0.085	0.1066
80	2.64	0.00535	23,200	13.66	22.7	1.07	0.0277	0.131	0.124	....	....
81	3.62	0.00815	23,200	31.10	25.5	1.01	0.0291	0.161	0.116	0.166	0.156
82	4.93	0.015	23,200	23.0	26.5	0.99	0.0296	0.219	0.116	0.266	0.212
83	1.74	0.00222	0	....	23	1.06	....	0.0842	....	....	....
84	2.64	0.00471	0	....	26.3	1.00	....	0.1227	....	....	....
85	3.67	0.00863	0	....	24	1.03	....	0.166	....	....	....
86	4.71	0.0136	0	....	26	1.00	....	0.2084	....	....	....
87	1.71	0.00199	100	....	24.3	1.03	....	0.0797	....	....	....
88	2.60	0.00476	100	....	23	1.06	....	0.1232	....	....	....
89	3.55	0.00808	100	....	24.7	1.02	....	0.1605	....	....	....
90	4.72	0.0136	100	....	26.5	1.00	....	0.2084	....	....	....
91	1.74	0.00222	300	0.004	23.3	1.05	....	0.0842	....	....	....
92	2.60	0.00466	300	0.065	21.5	1.11	....	0.1220	....	....	....
93	3.58	0.00823	300	0.125	24.8	1.06	....	0.1622	....	....	....
94	4.60	0.01327	300	0.142	22.0	1.09	....	0.206	....	....	....
95	1.715	0.00226	700	0.034	22	1.09	0.0584	0.085	....	....	....
96	2.59	0.00489	700	0.27	21	1.13	0.0573	0.125	....	....	....
97	3.56	0.00823	700	0.35	24.7	1.02	0.0612	0.1622	....	....	....
98	4.61	0.01322	700	0.44	23.0	1.06	0.0595	0.2058	....	....	....
99	1.73	0.00235	1,500	0.215	25.5	1.01	0.0622	0.0866	....	....	....
100	2.60	0.00484	1,500	0.868	22.8	1.07	0.0592	0.1243	....	....	....
101	3.52	0.0084	1,500	1.067	21.0	1.13	0.0572	0.164	....	....	....
102	4.57	0.0131	1,500	1.05	24.6	1.02	0.0611	0.2045	....	....	....
103	1.72	0.00244	3,100	0.385	25.0	1.02	0.0616	0.0884	....	....	....
104	2.59	0.00494	3,100	1.94	23.2	1.05	0.0596	0.1257	....	....	....
105	3.55	0.0085	3,100	2.21	21.5	1.11	0.0578	0.1648	0.121	0.167	0.1628
106	4.58	0.01323	3,100	2.16	24.7	1.02	0.0612	0.206	....	....	....
107	1.64	0.0026	6,300	0.45	23.0	1.06	0.0595	0.091	0.126	0.0902	0.092
108	2.58	0.00492	6,300	3.10	27.0	0.98	0.0638	0.1253	0.124	....	....
109	3.54	0.0 851	6,300	5.0	23.5	1.04	0.060	0.1649	0.124	....	....
110	4.59	0.01317	6,300	4.7	27.1	0.97	0.0638	0.205	0.119	0.209	0.202
111	1.64	0.00274	12,700	0.57	27.0	0.98	0.0638	0.0935	0.134	0.09	0.097
112	2.55	0.0054	12,700	5.1	24.5	1.02	0.0612	0.1313	0.127	0.1297	0.1332
113	3.50	0.00886	12,700	10.7	24	1.03	0.0607	0.168	0.124	....	....
114	4.60	0.01385	12,700	13.8	26.8	0.99	0.0636	0.2104	0.121	0.2126	0.2085
115	1.53	0.00318	25,500	1.3	22.5	1.088	0.0589	0.1008	0.148	0.091	0.110
116	2.58	0.00563	25,500	4.8	28	0.95	0.0648	0.1341	0.128	0.1322	0.1362
117	3.43	0.0096	25,500	13.5	26	1.00	0.0628	0.175	0.125	0.174	0.1762
118	4.47	0.0135	25,500	24.0	28	0.95	0.0648	0.208	0.124	....	....
119	1.44	0.0029	38,300	1.4	26	1.00	0.0628	0.096	0.154	0.083	0.107
120	2.56	0.0059	38,300	4.5	26.7	0.98	0.0636	0.1372	0.128	0.1343	0.1402
121	3.46	0.01005	38,300	13.0	24	1.03	0.0607	0.1794	0.126	0.1772	0.182
122	4.44	0.01425	38,300	30.0	25	1.02	0.0616	0.2135	0.125	0.212	0.215

[a] Dunes and streaks. [b] Not able

(Continued)

Universal Constant			Reynolds number, $R \times 10^{-5}$	Friction coefficient, $\lambda \times 10^{2}$	Ratio, $w/(kU_f)$		$z = w/(\beta k U_f)$		$\frac{\epsilon_{ss}}{\beta k y_m U_f}$	Bed condition[a]	Run
Top, k_t	Bottom, k_b	Average, k_{avg}			Top	Bottom	Top	Bottom			
(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(1)
0.341	0.313	0.327	6.55	1.98	0.97	1.03	0.562	0.65	0.224	....[a]	65
0.366	0.293	0.330	8.6	1.75	0.55	0.753	0.36	0.49	0.195	....[b]	66
0.347	0.299	0.323	12.2	1.68	0.476	0.574	0.318	0.372	0.20	....[b]	67
0.346	0.346	0.346	16.62	1.60	0.347	0.347	0.208	0.264	0.205	....[b]	68
0.386	0.30	0.343	5.67	2.26	0.786	0.93	0.482	0.593	0.207	Dunes	69
0.346	0.23	0.288	9.75	1.73	0.655	1.05	0.37	0.66	0.204	....[b]	70
0.370	0.283	0.327	12.7	1.63	0.452	0.634	0.30	0.39	0.203	....[b]	71
0.355	0.31	0.332	18.0	1.57	0.40	0.43	0.28	0.28	0.196	....[b]	72
0.343	0.260	0.301	5.9	2.42	0.87	1.07	0.56	0.62	0.233	Dunes	73
0.365	0.195	0.28	9.1	1.82	0.664	1.33	0.395	0.86	0.210	....[b]	74
0.360	0.216	0.288	10.35	1.69	0.52	0.99	0.30	0.655	0.174	....[b]	75
0.364	0.216	0.29	13.4	1.64	0.451	0.91	0.284	0.486	0.178	....[b]	76
0.346	0.256	0.301	16.2	1.60	0.436	0.653	0.265	0.40	0.188	....[b]	77
0.356	0.294	0.325	18.9	1.54	0.362	0.458	0.26	0.32	0.185	....[b]	78
0.350	0.285	0.318	6.08	2.8	0.95	0.936	0.61	0.62	0.226	....[b]	79
0.343	0.201	0.272	9.6	1.92	0.61	1.05	0.37	0.88	0.180	....[b]	80
0.326	0.221	0.274	13.9	1.56	0.538	0.843	0.263	0.56	0.181	....[b]	81
0.340	0.268	0.304	19.3	1.54	0.386	0.52	0.254	0.35	0.181	....[b]	82
....	....	0.373	6.40	1.84	....	....	....	....	....	....	83
....	....	0.367	10.2	1.70	....	....	....	....	....	....	84
....	....	0.368	13.9	1.60	....	....	....	....	....	....	85
....	....	0.37	18.4	1.53	....	....	....	....	....	....	86
0.360	0.348	0.354	6.45	1.70	....	....	....	....	....	Streaks	87
0.368	0.346	0.357	9.50	1.76	....	....	....	....	....	Streaks	88
....	....	0.356	13.6	1.60	....	....	....	....	....	Smooth	89
0.368	0.36	0.364	18.4	1.53	....	....	....	....	....	Smooth	90
0.38	0.364	0.372	6.45	1.83	....	....	....	....	....	Streaks	91
....	....	0.343	9.1	1.73	....	....	....	....	....	Streaks	92
0.366	0.349	0.357	13.2	1.61	....	....	....	....	....	Smooth	93
0.359	0.346	0.353	16.4	1.57	....	....	....	....	....	Smooth	94
0.390	0.376	0.383	6.12	1.92	....	....	....	....	....	Streaks	95
0.364	0.347	0.356	8.9	1.82	....	1.32	....	1.16	....	Streaks	96
0.363	0.328	0.346	13.6	1.62	....	1.15	....	0.85	....	....[b]	97
0.381	0.324	0.353	16.9	1.56	0.76	0.89	0.86	0.65	....	....[b]	98
0.338	0.333	0.336	6.66	1.96	....	2.14	....	1.63	0.242	Streaks	99
0.349	0.322	0.336	9.45	1.79	....	1.48	....	1.11	0.250	....[b]	100
0.362	0.337	0.350	12.2	1.69	0.96	1.03	0.83	0.80	0.226	....[b]	101
0.365	0.341	0.353	17.4	1.56	0.82	0.87	0.68	0.68	0.215	....[b]	102
0.376	0.351	0.363	6.55	2.06	....	1.98	....	1.57	0.40	Streaks	103
0.370	0.311	0.341	9.60	1.84	....	1.57	....	1.09	0.25	....[b]	104
0.372	0.30	0.336	12.4	1.69	0.93	1.18	0.72	0.94	0.223	....[b]	105
0.362	0.327	0.345	17.5	1.58	0.82	0.91	0.64	0.73	0.209	....[b]	106
0.376	0.353	0.364	6.01	2.41	....	1.83	....	1.47	0.42	Dunes	107
0.356	0.292	0.324	10.4	1.85	....	1.74	....	1.36	0.173	....[b]	108
0.348	0.265	0.307	13.2	1.70	1.04	1.37	0.31	1.07	0.236	....[b]	109
0.359	0.295	0.327	18.4	1.56	0.85	1.07	0.58	0.86	0.222	....[b]	110
0.385	0.262	0.323	6.50	2.55	....	2.51	....	1.73	0.53	Dunes	111
0.363	0.274	0.318	9.7	2.08	....	1.68	....	1.43	0.245	....[b]	112
0.368	0.21	0.289	13.2	1.80	0.98	1.7	0.67	1.21	0.23	....[b]	113
0.350	0.230	0.240	18.1	1.64	0.855	1.33	0.56	0.99	0.223	....[b]	114
0.388	0.272	0.330	5.5	3.40	....	1.97	....	1.67	0.54	Dunes	115
0.384	0.272	0.328	10.6	2.12	....	1.74	....	1.67	0.244	....[b]	116
0.380	0.215	0.297	13.3	2.04	0.95	1.65	0.78	1.28	0.208	....[b]	117
0.352	0.199	0.276	18.3	1.69	0.88	1.56	0.70	1.11	0.215	....[b]	118
0.337	0.272	0.305	5.23	3.62	....	....	....	....	0.59	....[b]	119
0.329	0.221	0.275	10.2	2.25	1.44	2.05	1.20	1.83	0.215	....[b]	120
0.374	0.187	0.280	13.1	2.10	0.92	1.78	0.82	1.37	0.193	....[b]	121
0.354	0.183	0.268	17.0	1.81	0.825	1.57	0.60	1.2	0.20	....[b]	122

to see the condition of the bed.

Like the objective of the experiments, the results can be divided into two parts. A third part will be given on the observations at the bed.

Effect of Suspended Sediment on the Characteristics of the Flow

The characteristics of the flow can be represented by three quantities: (*a*) von Kármán's universal constant k; (*b*) the momentum transfer coefficient ϵ_m; and (*c*) the coefficient of friction λ.

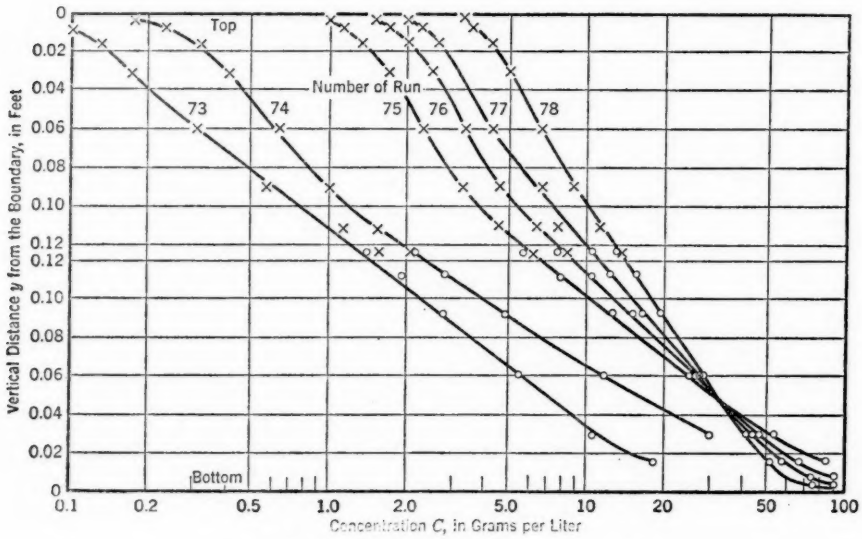


Fig. 7.—Sediment Concentration Profiles

von Kármán's Universal Constant, k.—The constant k can be calculated from the velocity defect law, Eq. 3, as follows: $\frac{u - u_{max}}{U_f} = \frac{2.3}{k} \log \frac{y}{y_m}$; that is, the slope of the semilogarithmic velocity profile will give k/U_f. Then

$$K = 2.3\ U_f \times \text{slope of the semilogarithmic velocity profile} \ldots\ldots (9)$$

but

$$\overline{U}_f = \sqrt{\frac{\tau_o}{\rho}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which τ_o equals the shear at the point of intersection of the profile with the boundary. Thus, in order to find k, τ_o must be known.

By using the experimental data of M. Bazin, Garbis H. Keulegan[12] showed that the value of the shear changes along the bottom of rectangular channels.

He found that the values of U_f at the center of the bed, divided by the average $\overline{U}_f$ varied from 1.00 to 1.07 for moderate roughness. He did not give any

[12] "Laws of Turbulent Flow in Open Channels," by Garbis H. Keulegan, *Journal of Research*, National Bureau of Standards, U. S. Dept. of Commerce, Vol. 21, 1938, pp. 707–741.

data for smooth channels. A few measurements, in the present work, were made covering the entire section—runs 1, 2, 5, 6, and 7 for clear water and run 71 for loaded water. Runs 1, 2, 5, and 6 showed that U_f was uniform along the boundary, whereas in run 7, the ratio $U_f/\overline{U}_f$ at the center was 1.04. The distribution of U_f is an interesting problem and needs more study. This distribution might have been responsible for the small variation in the values of k found for clear water in channels of different shapes. However, it was clear that the distribution of U_f could contribute to the variation in the slopes of the semilogarithmic velocity profiles, and, hence could introduce errors into the proposed method of computing k. Its effect, however, seemed to be very small with respect to the values observed here (k decreased as much as 50%). In the present work U_f was assumed to be uniform along each wall.

To find the shear velocity at top and bottom (U_{tf} and U_{bf}), the flow was assumed to be two dimensional and, therefore, $d\tau/dy = dP/dx$, in which p is the pressure intensity and x is the horizontal distance along the axis of the channel. Assuming a uniform pressure gradient in the direction of flow, τ will be linearly related to y (Fig. 8)—that is, $\tau_{ot} = \tau_{ob}\,(y_{tb}/y_m - 1)$, in which y_{tb} is the total depth from top to bottom boundary; τ_{ot} is the intensity of shear at the top wall; and τ_{ob} is the intensity of shear at the bottom wall. (The symbol y_m is the coordinate of the point of zero shear or the point of maximum velocity.) Also, $\frac{1}{2}(\tau_{ob} + \tau_{ot}) = \tau_o$ in which τ_o is the average intensity of shear over the entire boundary. Therefore, $\tau_{ob}\, y_{tb}/y_m = 2\,\overline{\tau}_o$, and $\sqrt{\tau_{ob}} = \sqrt{\overline{\tau}_o}\,\sqrt{2\,y_m/y_{tb}}$ or $U_{fb} = \overline{U}_f\,\sqrt{2\,y_m/y_{tb}}\,\sqrt{\rho \text{ average}/\rho \text{ bottom}}$.

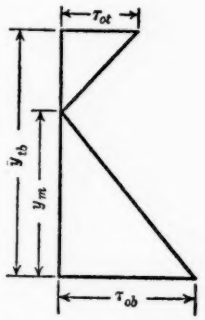


FIG. 8.—SHEAR DISTRIBUTION

For a concentration of 33 g per liter, the ratio of the density for loaded water to that of clear water was 1.02 and its square root was 1.01; that is, it would cause an error of 1%, which is within the experimental error. For concentrations less than 30 g per liter the correction for the density was neglected; that is, $U_{fb} = \overline{U}_f\,\sqrt{2\,y_m/y_{tb}}$ and $U_{ft} = \overline{U}_f\,\sqrt{2\,(1 - y_m/y_{tb})}$.

These values are given in Cols. 11 and 12, Table 1. The values of k which were computed from these values and the slopes of the (u- versus-log y)-lines, are given in Cols. 13 and 14. For both sands k decreased from 0.373 to about 0.20 as the average concentration increased.

Fig. 9 shows the relation between the values of k and the total load of sand, W, in the system for two mean velocities in the channel. The points were somewhat scattered but Fig. 9 shows the definite tendency of k to decrease with the increase in the load. For the same total loads Fig. 9 shows that the 0.16-mm sand had more effect on k at the bottom than the 0.10-mm sand. This is not true because, for the same total load and for the same speed, concentration at the bottom is higher for the 0.16-mm sand than for the 0.10-mm sand; and so it can be assumed that both sands have nearly the same effect on k.

Moment Transfer Coefficient ϵ_m.—Some of the semilogarithmic plots of velocity profiles are given in Fig. 5. The experimental points follow reasonably well the logarithmic law over most of the section. This proves that ϵ_m still

follows Eq. 4*a* for all those different concentrations. The only changes in ϵ_m will be due to changes in k.

The central region should be excluded from this rule. Messrs. Brooks and Berggren[7] showed that a very slight change in fitting the Nikuradse experiments will cause an appreciable difference in the values of ϵ_m. For example, a constant ϵ_m will give a second-degree parabola for the velocity profile, and this parabola can fit the velocities reported by Mr. Nikuradse for $R_e = 43{,}400$ within 0.6% over the middle half of the section. It is difficult to evaluate ϵ_m at this region from velocity measurements, because $\epsilon_m = \dfrac{\tau}{\rho\, du/dy}$ and both τ and du/dy approach zero at the center. For this reason, no attempt is made at this stage to evaluate ϵ_m from the velocity profiles. It will be evaluated indirectly by studying ϵ_s and the relation between ϵ_m and ϵ_s.

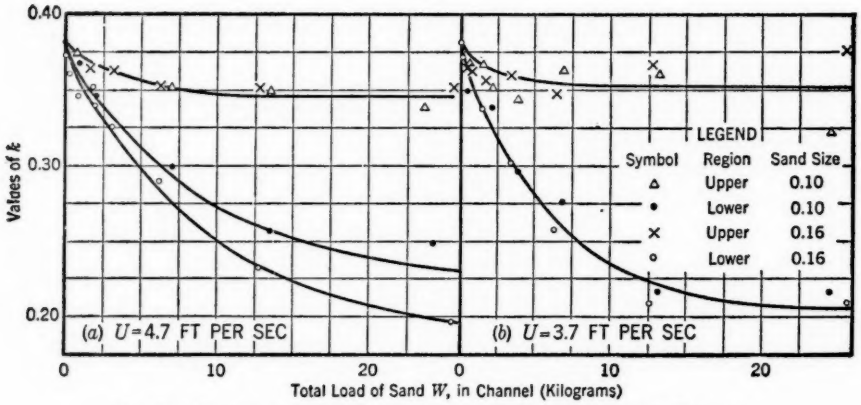


FIG. 9.—CHANGE OF k WITH THE TOTAL SAND LOAD, W, IN THE CHANNEL

It can be observed from Fig. 5 that there was a deviation from the logarithmic law at high concentrations near the bed of the stream. This occurred in Mr. Vanoni's[8] work.

Coefficient of Friction, λ.—Mr. Keulegan[12] furnished the relation between the friction coefficient λ and the Reynolds number $\mathbf{R}$ for a smooth rectangular channel:

$$\sqrt{8/\lambda} = a_s - 2.25 + \frac{2.3}{k} \log \left(\frac{\mathbf{R}\sqrt{\lambda}}{4\sqrt{8}} \right) \ldots\ldots\ldots\ldots\ldots (11)$$

In Eq. 11, $a_s = N - 2.3/k \log N$; $\mathbf{R} = 4\, r_h\, U/\nu$; and N is the characteristic value of the parameter $U_f\, y/\nu$, to which the transition between the boundary laminar film and the inner turbulent region corresponds.

Taking $N = 11.6$ according to the experiments of Mr. Nikuradse, and $k = 0.373$ according to Table 1, runs 1 to 7, Eq. 11 becomes

$$\frac{1}{\sqrt{\lambda}} = -1.32 + 2.18 \log \mathbf{R}\sqrt{\lambda} \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Fig. 10 shows such a curve with the experimental results. The curve fitted the points of the clear water quite satisfactorily. This proves that the walls

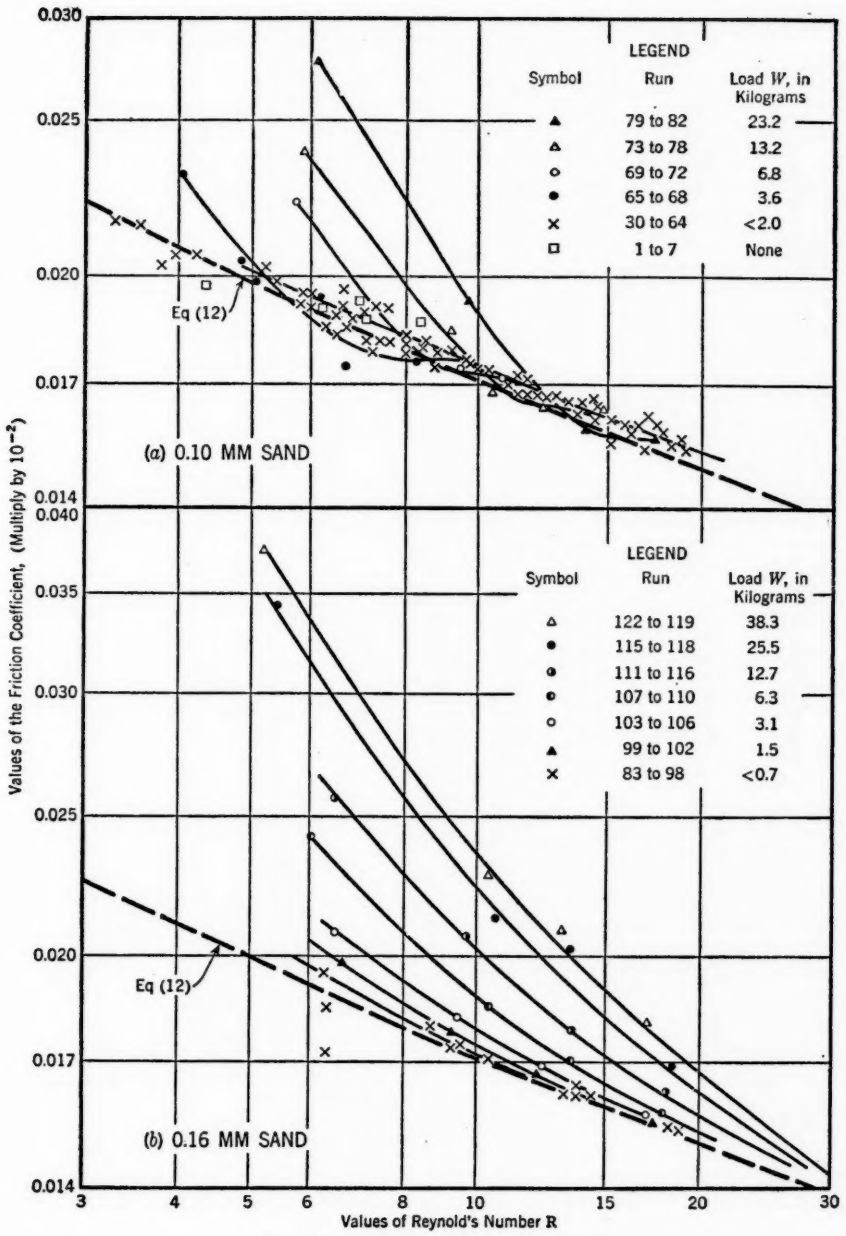


FIG. 10.—EFFECT OF SEDIMENT ON λ

are smooth. The general pattern of the results plotted in Fig. 10 agrees very well with the results collected by Mr. O'Brien and R. G. Folsom.[13]

Taking the curve λ versus **R** of Eq. 12 as a reference, it can be observed that, for each concentration of each sand, there was a certain critical Reynolds number below which the friction coefficient began increasing with respect to that of the clear water, and above which the coefficient of friction for all concentrations was practically the same. It was observed from the glass windows that the critical value of **R** corresponded to the velocities that were able to

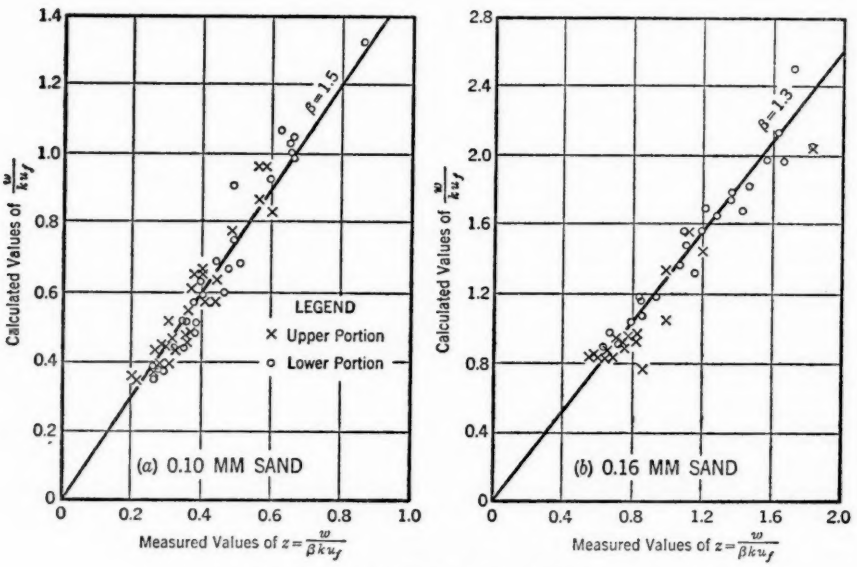


FIG. 11.—EVALUATION OF THE FACTOR β

sweep all the dunes from the floor. Below these critical values of **R** an appreciable fraction of the sand was moving as a bed load in dune forms, and above these values of **R** there were no dunes and most of the sand was carried in suspension as clouds. This fact was also observed from the values of $\bar{c}$ given in Col. 5, Table 1 and from Fig. 6. Above these critical values of **R** it was difficult to increase $\bar{c}$ since most of the sand was already in suspension. It just redistributed itself, as shown by the crossing and overlapping of the concentration profiles (Fig. 6).

SEDIMENT DISTRIBUTION AND SEDIMENT TRANSFER COEFFICIENT

The two sets of curves given in Figs. 6 and 7 show the sediment distribution over the section. Fig. 6 was drawn to verify Eqs. 7, whereas Fig. 7 was drawn to verify Eq. 6.

For most of the depth of the channel (excluding the central part) Fig. 6 shows that Eqs. 7 hold well. The slight curvature at the top wall will be

[13] "The Transportation of Sand in Pipelines," by M. P. O'Brien and R. G. Folsom, *Publications in Engineering*, Univ. of California, Berkeley, Calif., 1937, Vol. 3, No. 7, pp. 343–383.

discussed subsequently to prove that this deviation is due to sand grading. According to Eqs. 7, the slope of the lines of log C and log h must be equal to z. The values of z measured by this method are given in Cols. 20 and 21, Table 1, whereas the calculated values of $w/k\ U_f$ are given in Cols. 18 and 19. The upper and lower halves of the channel were treated separately. Fig. 11 shows the relation between the measured values of z (which should be $w/(\beta\ k\ U_f)$) and the values of $w/(k\ U_f)$. The linear relation in both curves for the two sands shows that the assumption of Eq. 4 is fairly reasonable. For the 0.10-mm sand, $\beta = 1.5$; and, for the 0.16-mm sand, $\beta = 1.3$.

Fig. 7 helps in studying the central part of the flume. The straight parts of the curves (which always occur in the center) indicate that the assumption of a constant ϵ_s here is quite satisfactory. The constant value of ϵ_s in the central parts of the flow (which will be called ϵ_{sc}), can be evaluated directly from Eq. 6 and from these curves. The nondimensional values $\epsilon_{sc}/(\beta\ k\ y_m\ U_f)$ are given in Col. 22, Table 1. This value is nearly constant for most of the runs, for both the sands, and its average value is about 0.22. The only runs that are far from

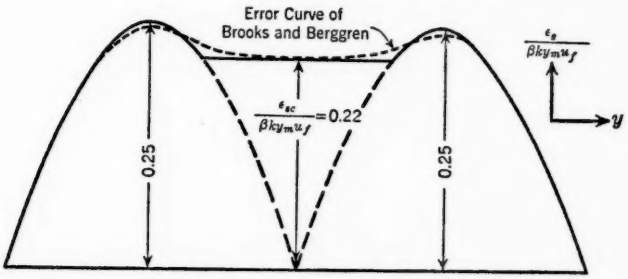


FIG. 12.—DIMENSIONLESS FORM OF ϵ_s OR ϵ_m

this value are those at low speed and high concentration. In the latter runs big dunes were observed and the conditions of the flow were believed to be unsteady because of the periodical disturbance caused by the travel of dunes past the measuring point, so these readings were excluded. The straight part, and accordingly the constant ϵ_{sc}, was observed to cover at least the middle third of the channel depth in each run (Fig. 7). When ϵ_{sc} was measured and compared with the parabolic form of Eq. 4*b*, it was found that it would intersect it at the third points, as in Fig. 12. According to the present experiment, Fig. 12 would be the most reasonable approximation of the dimensionless form of ϵ_s. As previously mentioned, it could also represent the form of ϵ_m.

BED LOAD

The bed load was observed through the glass windows. For high speeds and high concentrations it was not possible to see what was occurring. For small loads of sand and for low velocities the bed load was in the form of streaks. These streaks were not very steady and their particles were moving in steps. When the velocity was increased, all the streaks disappeared and the particles went into suspension.

When the concentrations were somewhat increased for low speed, dunes began to appear on the bed as isolated individual dunes of a crescent shape, small in size compared to the channel width. At the beginning, a single dune might appear every 5 ft or 6 ft along the bed. When the concentration was increased, the number of those dunes increased and they started joining each other to form one dune that extended across the channel. The form of the bed load is described by letters in Col. 23, Table 1.

A new phenomenon was observed for the maximum loads reached in the case of the 0.16-mm sand. During run 119, which was made with the low velocity of 1.7 ft per sec, very large dunes or waves were observed on the bed. They occurred irregularly every now and then; sometimes two might follow each other closely and sometimes more than half an hour elapsed between these waves. A wave was about 20 in. long and 0.6 in. in maximum height and would cover the full width of the flume. The dunes and waves of sand in that run were so large and unsteady that the entire flow was considered unsteady; then the results of the run were rejected.

With the same concentration but with higher velocities it seemed as if the dunes and waves were smoothed out and that a uniform layer was moving along the floor with small dunes on its surface. In run 120 this layer was about 0.25 in. thick, and the dunes were about 0.05 in. high over the layer.

Discussion of Results

Effect of Suspended Load on the Flow.—The results of the studies can be appraised by a discussion of the effect of the suspended load on four characteristics of the flow: (*a*) the universal constant; (*b*) the momentum transfer coefficient; (*c*) the friction factor; and (*d*) the velocity distribution.

(*a*) The Universal Constant, k.—In his similarity theory of turbulence pattern Mr. von Kármán[3,6] introduced k as the factor of proportionality between the velocity derivatives and the mixing length $\frac{du/dy}{d^2u/dy^2} = l/k$, in which l is the Prandtl mixing length.

Assuming that the turbulence pattern will remain similar from point to point, k will remain constant over the cross section of the flow. The decrease in the value of k, which was observed by Mr. Vanoni, varied from run to run, but in each run k was the same for the entire profile. In the present work, k for the upper part of the section was constant; and k for the lower part was also constant but had a different value than for the upper part. This means that the change in k was not local from point to point following the different concentrations at these points, but changed from section to section and remained constant over each section. It seems, then, that the presence of a certain proportion of sediment in suspension will affect the main pattern of the turbulence in the entire region from the controlling boundary to the zero shear plane.

The slight curvature in the semilogarithmic velocity profiles observed in the case of high concentration near the bed (Fig. 5) cannot be explained with any local change in k. If k is affected by the local concentration, it should decrease as the bed is approached and this will curve the line in the opposite direction to that shown in Fig. 5. This curvature was observed to occur only near the

bottom for appreciable concentrations and it increased with the increase in the load. This might be evidence that the curvature is due to some interaction between the flow and the presence of the particles at the bed. The question of the local effect of sand on k has been raised by E. R. Van Driest.[14]

By measuring the velocity profiles of the wind above a circular concrete pavement, P. A. Sheppard[15] observed an approximate variation in the values of k from 0.6 to 0.2 in the lower atmosphere. The constant of turbulent transfer decreases as the atmospheric stability increases. Mr. Sheppard declared these values are very rough; the wind profiles at the experimental site were not representative of the boundary layer because of the limited extent of the measuring surface (concrete.)

The only source of error in evaluating k can be due to a large variation in the U_f-distribution along the boundary. Run 71 showed that the existence of sand could never change the U_f-distribution to take care of even a small part of the changes in the semilogarithmic velocity profile slopes.

(*b*) Momentum Transfer Coefficient, ϵ_m.—It has been shown how ϵ_s was calculated and how Fig. 12 represents its distribution. The relation (Eq. 2) between ϵ_m and ϵ_s was verified in the part of the section where both ϵ_m and ϵ_s could be measured directly. It is very reasonable to assume that this relation will hold over the remainder of the section—that is, the central part, where ϵ_m is not known. Therefore, Fig. 12 can represent the distribution of ϵ_m as well as ϵ_s. This curve agrees very well with the curve given by Messrs. Brooks and Berggren[7] under their assumption of the error curve (see Fig. 12). The present relation is easier to handle.

(*c*) The Friction Factor, λ.—The results of the present work as given in Fig. 10 agree very well with the results reported by Messrs. O'Brien and Folsom.[13] The findings of G. W. Howard,[16] M. ASCE, when represented graphically (as recommended by Josef E. Montgomery[17]) agree with the other results. Below certain Reynolds numbers λ increases with the concentration, whereas, for greater values the concentration has no effect. Comparing the different curves of λ versus $\mathbf{R}$ given in this paper and elsewhere,[13,17] it is clear that the curves of the different concentrations approach the clear water curve in different ways which depend mainly on the size of the suspended material. Messrs. O'Brien and Folsom[18] show how the resistance of the clay slurry deviates sharply from the clear-water curve. This is similar to Fig. 10(*a*) for the 0.10-mm sand. When the size of the sand was increased to 0.16 mm (Fig. 10(*b*)) the curves approach the clear water more smoothly over a longer region.

From the careful observation of the bed in the present work, these critical velocities are related to the existence of the dunes on the bed. It is clear that for the same load and velocity the bed load will increase with the size of the

[14] Discussion by E. R. Van Driest of "Transportation of Suspended Sediment by Water," by Vito A. Vanoni, *Transactions*, ASCE, Vol. 111, 1946, p. 106.

[15] "The Aerodynamic Drag of the Earth's Surface and the Value of von Kármán's Constant in the Lower Atmosphere," by P. A. Sheppard, *Proceedings*, Royal Soc. of London, January, 1947, p. 208.

[16] "Transportation of Sand and Gravel in a Four-Inch Pipe," by G. W. Howard, *Transactions*, ASCE, Vol. 104, 1939, pp. 1334–1380.

[17] Discussion by Josef E. Montgomery of "Transportation of Sand and Gravel in a Four-Inch Pipe," by G. W. Howard, *Transactions*, ASCE, Vol. 104, 1939, p. 1350, Fig. 9.

[18] "The Transportation of Sand in Pipelines," by M. P. O'Brien and R. G. Folsom, *Publications in Engineering*, Univ. of California, Berkeley, Calif., 1937, Vol. 3, No. 7, p. 361, Fig. 9.

suspended material. It is also clear that the chances of forming dunes increase with the increase of the bed load. This is the reason for the dependence of the critical Reynolds numbers on the size of the suspended material. The initial roughness might affect these critical numbers slightly. If a channel has very rough walls, say, in the case of Fig. 10, it can be expected that the reference line of clear water will be raised. There will still be a transition region, but the critical values might be reached at smaller values of **R** than in the case of a smooth wall.

With the indicated substitutions, Eq. 11 for smooth channels becomes

$$\sqrt{\frac{8}{\lambda}} = \frac{N - 2.3}{k \log N} - 2.25 \frac{+ 2.3}{k \log \frac{r_h U \sqrt{\lambda/8}}{\nu}} \quad \ldots\ldots\ldots\ldots (13)$$

in which N is represented in a Reynolds-number form for the thickness of the laminar sublayer δ, as follows: $U_f \delta/\nu = N$. It was found empirically that $N = 11.6$ for the clear fluids. For rough walls λ follows the relation:

$$\sqrt{\frac{8}{\lambda}} = -\frac{1}{k} + \left(\frac{U_w}{U_f} - \frac{2.3}{k} \log \frac{y_m}{e} \right) + \frac{2.3}{k} \log \frac{y_m}{e} \quad \ldots\ldots\ldots (14)$$

in which e is the equivalent roughness height parameter and U_w is the wall velocity.

Two main factors will affect λ. One factor is the internal system of flow represented by k and can be called the internal factor. The other one is the boundary conditions represented by either δ and ν if the walls are smooth (or by the roughness height e if they are rough); and this can be called the boundary factor. There are some other factors, but they appear in terms of secondary order of magnitude and so can be neglected.

Varying the internal factor by varying k alone and holding the boundary factor constant, λ will change approximately as the square of k, according to Eq. 14. It must decrease in the presence of the suspended material, because of the decrease in k. As for the effect of the boundary factor, it can be seen that with a sediment-laden flow and a continuous interchange of particles between the bed and the fluid, it is quite difficult to visualize the picture at the boundary. It is conceivable that the laminar sublayer will be broken up by the continuous bombardment of the sand particles through it. This will increase λ and the bottom region will be somewhat similar to the case near a rough surface even if it looks smooth. When there are dunes at the bed, it is obvious that they greatly increase the bed roughness and λ will increase. Now when both the factors are considered, λ will decrease or increase according to the ratio between the effects of the two factors.

At low velocities, below the critical value, the dunes will increase the boundary effects and accordingly λ will increase, as seen from Fig. 10. Just after the dunes are swept off the bed, in the case of the fine particles, it seems as if the effect of k exceeds the boundary-layer effect of the sediment at the bottom and λ is less than for the clear water. This can be seen in Fig. 10(a). At high velocities the two effects are nearly balanced and λ is not changed.

(*d*) Velocity Distribution.—Fig. 13 shows some of the velocity profiles at the center line, for high concentrations, compared to the profiles of clear water flow, which have nearly the same values of U.

Fig. 13(*a*) shows the effect of dunes on the velocity distribution. It shows how the maximum velocity and, correspondingly, the plane of zero shear were shifted upward. This indicated that the shear at the bed was increased (Fig. 8) causing λ to increase with the dunes. When the effect of the dunes was not so strong the other factors came into the picture and resulted in profiles such as Figs. 13(*b*), 13(*c*), and 13(*d*).

For high velocity it was shown in Fig. 10 that λ (or u/U_f) was the same for both clear and sediment-laden water. Fig. 13(*d*) and Col. 10, Table 1, showed that in these cases the maximum velocity always lies at the center; therefore, U_{tf} at the top must equal U_{bf} at the bottom. There is no other way to explain the big difference between the velocity profile slope at the top half and the bottom half except by the change in k. It is also of interest to compare the two sides of the velocity profile for high concentration (see Fig. 13(*d*)). The velocity near the bottom shows quite a decrease in the wall velocity U_w exactly as if the roughness of that wall had been increased. This could be explained by the effect of the sand in breaking up the laminar sublayer and decreasing its thickness in the same way as increasing its roughness.

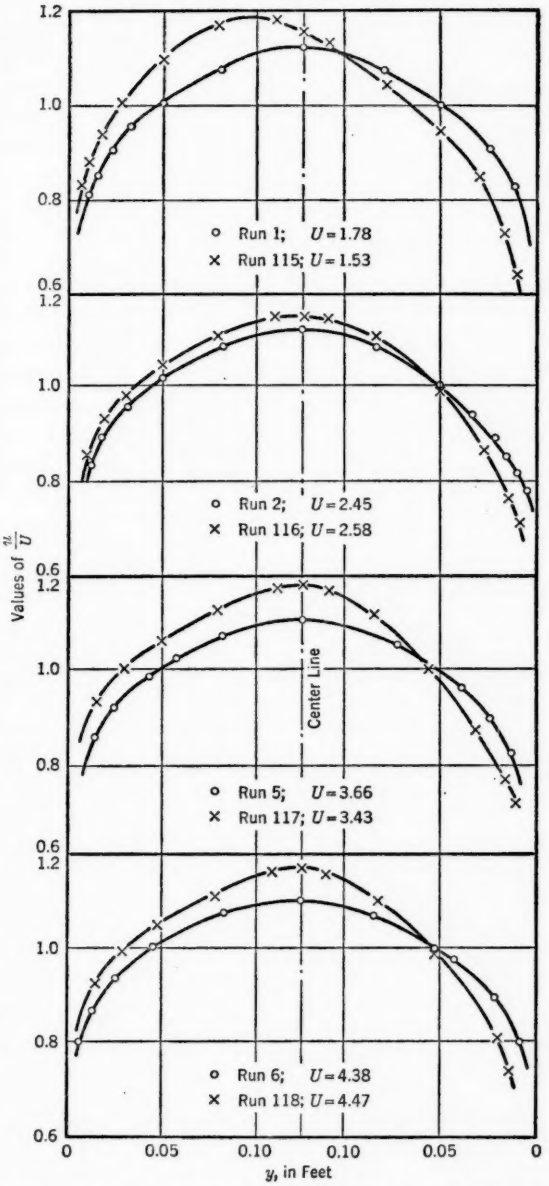


FIG. 13.—EFFECT OF SUSPENDED SAND ON VELOCITY PROFILES

Sediment Distribution.—The sediment concentration curves of Fig. 6 are slightly curved near the upper surface of the channel. The concentrations in this region were usually very low; and, when some of the samples were examined carefully by eye, they appeared to be finer than the sand customarily used. It is known that however carefully the sand is sieved it will always have a certain standard deviation of particle sizes, as shown in Fig. 4. Dividing the sample

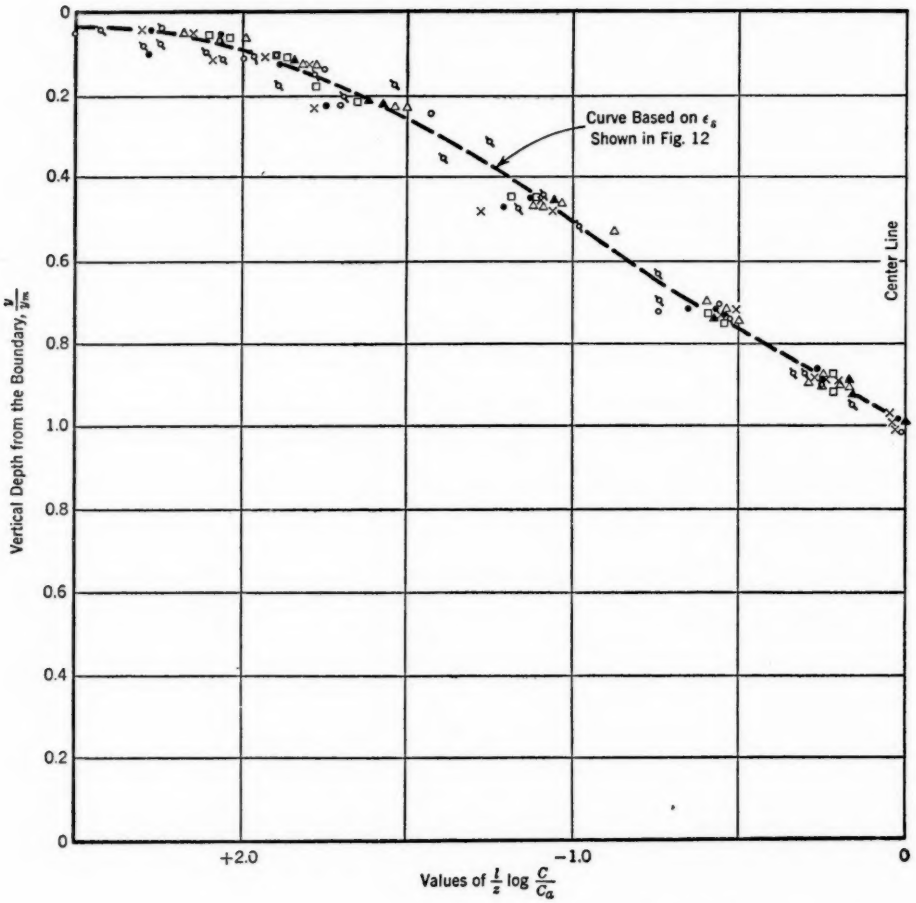
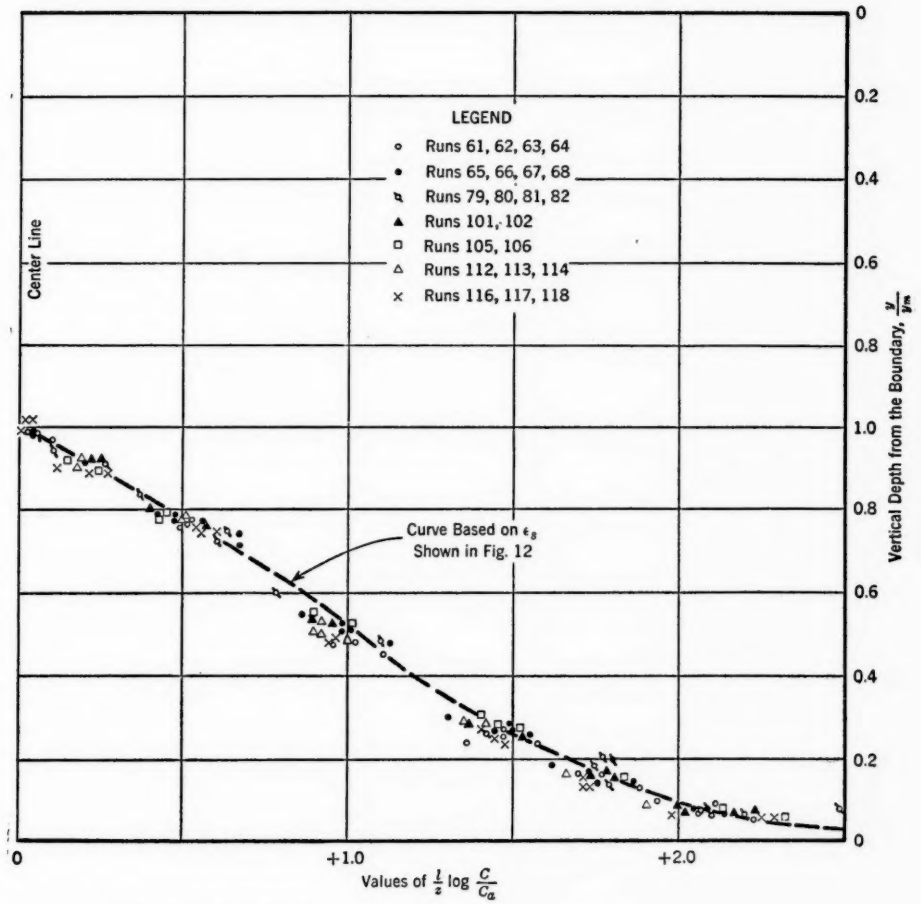


FIG. 14.—DISTRIBUTION OF THE

into different sizes, it is clear that the very fine portion will be more uniformly distributed than the coarser portions. Now, knowing that Fig. 6 is for log C and that the concentration is very low at the top, it is clear that the small proportion of the very fine uniformly distibuted particles will amount to an appreciable fraction of the sample at the top, whereas it is negligible at the bottom. This grading is then the cause of the observed curvature.

It was shown in the results that $\epsilon_s = 1.5\ \epsilon_m$ for one sand and $\epsilon_s = 1.3\ \epsilon_m$ for the other sand. These two observed values are not sufficient to establish the relation between β and the size of sand. More sizes and mixtures of sizes should be tested in the future. Messrs. Sherwood and Woertz[4] found the eddy diffusion in the main body of the turbulent fluid for the case of vaporization of water into a turbulent gas stream to be equal to $1.6\ \epsilon_m$. W. Corcoran[19] found



SEDIMENT CONCENTRATION ($a = y_m$)

that ϵ_c, the heat transfer coefficient between two parallel plates, varied between 1.5 and 1.22 ϵ_m with an average of $1.3\ \epsilon_m$.

Fig. 12 represents the assumed distribution of the nondimensional form of ϵ_s or ϵ_m across the central profile of the channel. Fig. 14 shows a curve drawn

[19] "Temperature Gradients in Turbulent Air Streams," by W. Corcoran, thesis presented to the California Inst. of Technology at Pasadena, in 1948, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

for the sediment concentration distribution under this assumption. It must be noticed that the measured values of k were always used for plotting the points in this curve. The points of twelve runs for the 0.10-mm sand and ten runs for the 0.16-mm sand are shown in Fig. 14. The agreement between the experimental results and the assumed theoretical curve is remarkable, which proves that the assumption in Fig. 12 is reasonably correct.

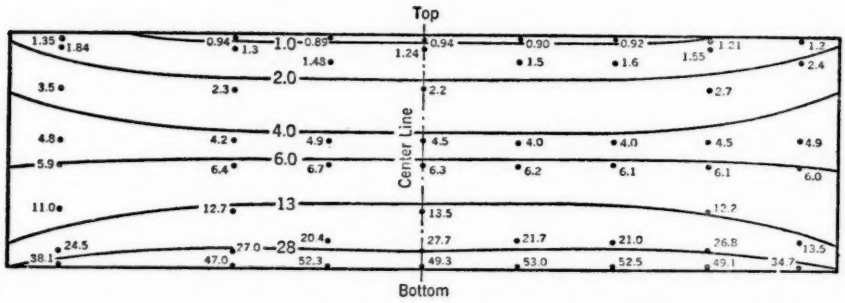


FIG. 15.—SEDIMENT CONCENTRATION DISTRIBUTION OVER THE SECTION FOR RUN NO. 71 (GRAMS PER LITER)

Fig. 15 shows the concentration distribution over the entire cross section for run 71. It is clear that in the middle half the contours run parallel to the top and bottom, showing that the assumption of a two-dimensional flow is very reasonable. Near the side walls the concentration is more uniform over a vertical section, which indicates that ϵ_s is stronger near the side walls than it is in the center. Mr. Van Driest[14] has explained the more uniform sediment concentration near the wall by saying that ϵ_s in any vertical line is more uniform near the wall. This could not be the reason because, as seen from Fig. 7, the average slope of the concentration profile was mainly controlled by the average value of ϵ_s at the profile and not by its variation.

CONCLUSIONS

1. The von Kármán constant k decreases when the load in suspension is increased. Both of the sands used in the experiments have nearly the same effect on k, which decreased to as low as 0.20 for a load of sand equal to 25 kg., in other words, when the sand content of the system was 43 g per liter. The decrease of k indicates damping of the turbulence.

2. The change in k does not follow the change in the concentration from point to point over one cross section, but it varies from section to section, maintaining a constant value over each one. This value decreases with the increase of the total quantity of suspended sand in the section.

3. The value of the momentum transfer coefficient is affected by the presence of the sand only through the changes in k. The form of its distribution curve will remain the same for sediment-laden water as for clear water. This coefficient cannot be measured directly at the center of the channel but, because it is similar to ϵ_s, one can reasonably assume that it follows Fig. 12.

4. The coefficient of friction for a stream carrying suspended sediment exceeds that for clear flow only when dunes form on the bed. If there are no dunes, the present experiments show that λ can hardly be affected by the presence of sand. This is for smooth walls and probably for walls roughened by particles smaller than, or equal to, the suspended particles.

5. The sediment transfer coefficient ϵ_s follows Fig. 12. It is approximately equal to 1.5 ϵ_m for the 0.10-mm sand and 1.3 ϵ_m for the 0.16-mm.sand.

6. Two devices—the basic equation for sediment distribution Eq. 1 and the distribution curve of ϵ_s in a vertical profile in the channel (see Fig. 12)—can define, accurately, the concentration distribution as represented in Fig. 14. The only difficulty in this procedure is that k and β cannot be predicted from the theory and they must be assumed or determined experimentally.

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APPENDIX. NOTATION

The following letter symbols, introduced in this paper, are assembled for the convenience of discussers:

a = a reference level; $a_s = N - \dfrac{2.3}{k \log N}$;

C = concentration of suspended material, expressed as mass per unit volume; C_a denotes a value of C at an arbitrary reference level;

c = concentration over a given cross section; $\bar{c}$ denotes a measured mean value of c;

D = diameter of sand particles:

D_m = geometric mean sieve size;

D_s = sedimentation diameter of sand;

e = an equivalent roughness-height parameter;

g = acceleration of gravity;

$h = \dfrac{y_m}{y} - 1$; $h_a = \dfrac{y_m}{a} - 1$;

i = hydraulic gradient slope;

k = von Kármán universal constant of turbulent exchange:

K_b = the constant k in the bottom region of a section;

K_t = the constant k in the top region of a section;

l = mixing length;

N = the characteristic value of the parameter $\dfrac{U_f \delta}{\nu}$;

P = pressure intensity;
$\mathbf{R}$ = Reynolds number;
r_h = hydraulic radius;
t = temperature;
U = velocity of flow; mean velocity over the cross section:
- U_b = value of U at the bottom wall;
- U_f = the Prandtl friction velocity = $\sqrt{\frac{\tau_o}{\rho}}$;
- U_t = value of U at the top wall or boundary;
- U_w = value of U at the wall of a flume or a channel;

u = velocity (see also U):
- u' = turbulent velocity;
- u'' = average velocity along the center-line profile;
- $u_{\max}$ = maximum value of u;

W = total load of sand in the flume or channel;
w = settling velocity of a particle in still fluid;
x = horizontal distance along the axis of the channel;
y = vertical distance from the boundary:
- y_m = y measured from the boundary to the plane of maximum velocity, also, the coordinate of the point of zero shear;
- y_{tb} = the distance y from the top boundary to the bottom boundary;

$z = \frac{w}{\beta k U_f}$;
β = factor of proportionality;
δ = thickness of a laminar sublayer;
ϵ = transfer coefficient:
- ϵ_m = momentum transfer coefficient;
- ϵ_s = sediment transfer coefficient;
- ϵ_{sc} = value of ϵ_s at the center of the profile;

λ = friction coefficient;
ν = kinematic viscosity;
ρ = density of fluid; mass per unit volume;
σ = standard deviation; σ_g = geometric standard deviation; and
τ = shearing stress:
- τ_o = value of τ at the point of intersection of the profile with the boundary;
- τ_{ob} = value of τ at the bottom boundary;
- τ_{ot} = value of τ at the top boundary.

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